Passivity-based Coordination of Multi-Agent Systems: A Backstepping Approach

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Abstract—In this article we extend existing results for the passivity-based design of cooperative control laws to systems with arbitrary relative degree. To this end, we exploit the feedback equivalence of strict feedback systems to passive systems and design the controls based on an alternative backstepping procedure. This results in a virtual output, exchanged by the agents and synchronization of the group. Given a static strongly connected and balanced communication network we prove synchronization of the group with the proposed controller using the Krassovskii-LaSalle invariance principle. The effectiveness of our methodology is illustrated in numerical simulations, solving the rigid body attitude synchronization problem.

I. INTRODUCTION

Cooperative control among autonomous agents has attracted a great deal of interest within the past decade. Applications such as environmental monitoring with unmanned vehicles, automated highway systems, and spacecraft clustering have greatly influenced recent developments in control theory. Problems of interest include formation control, consensus, coverage and distributed optimization [14],[16],[17].

A number of different approaches have been suggested for designing controllers that enable a prescribed group behavior. Although the early focus was on centralized approaches, the emphasis today is on decentralized and distributed control to ensure computational efficiency, robustness to loss of agents, and so on. Many of the existing methods, such as virtual structure approaches [13], lack sufficient generality to be broadly applicable.

In contrast, [1] and [3] presented a unifying approach to design and analyze coordination control laws in cases where the group members are passive systems [23]. Also of interest is the earlier work [18], which considered synchronization of chaotic oscillators. The general findings of [1] and [3] have recently been used to design passivity-based coordination controllers for nonlinear systems in [8],[9]. Crucial, as for all decentralized coordination strategies, is the dependency of the control laws on the exchange of information within the group. The communication network topology affects the dynamics of the aggregate motion of the group and the properties of this network must be examined [10],[17].

An advantage of passivity-based coordination is its delay independence, i.e. synchronization is not corrupted by delays in the communication.

In this work, we want to extend the methodology of "coordination by passivation" and remove the relative degree obstacle in the design of cooperative control laws, left open in [3]. Given systems in strict feedback form [12], it has been shown in [21] that these are feedback equivalent to passive systems. Moreover, as a simple cascade of integrators, these systems can be stabilized through input-state feedback linearization or through backstepping [12].

Our goal is to combine feedback passivity and an alternative backstepping procedure to derive decentralized control laws coordinating a group of agents whose dynamics are given in strict feedback form. Hence, the results of [4] are extended to systems having an arbitrary relative degree. Because the controllers are decentralized, they rely on their own system model and on information exchange within the group, and they are individually tunable to improve performance. Moreover, because the passivity property is preserved in the closed-loop system, the proposed controller enjoys the same properties encountered in [1] and [3].

The paper is organized as follows: In Section II, we introduce a system description and the necessary definitions for passivity and graph theory relevant to our problem. In Section III, we present our general derivation of the synchronization control law and state our main result in Theorem 2. Finally, we illustrate the effectiveness of the methodology in an example solving the attitude coordination problem of rigid bodies in Section IV. Conclusions are provided in Section V.

II. PRELIMINARIES

In this Section we will briefly introduce the mandatory definitions and notions used in the subsequent passages.

A. Passivity

Passivity as a system property was introduced in [24],[25] and can be seen as an energy conservation aspect of a dynamic system. For a thorough introduction of this concept, we refer to [23] and the early publications [7],[15].

Consider the system

\[ \dot{x} = f(x) + G(x)u, \]
\[ y = h(x), \]

where \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^m \) and \( y \in \mathbb{R}^m \). The functions \( f(\cdot) \in \mathbb{R}^n, g(\cdot) \in \mathbb{R}^{n \times m} \) and \( h(\cdot) \in \mathbb{R}^m \) are assumed to be sufficiently smooth and we assume that \( f(0) = h(0) = 0 \). The inputs
\textbf{Definition 1:} The nonlinear system (1) is said to be passive if there exists a scalar $C^1$ storage function $V(x) \geq 0$, $V(0) = 0$ and a function $S(x) \geq 0$ such that for all $t \geq 0$
\[ \dot{V}(x) = u^T(t)y(t) - S(x(t)). \]
It is called strictly passive if $S(x) > 0$ and lossless if $S(x) = 0$.

This is known as the differential condition for passivity of system (1). Note that even a positive semidefinite storage function $V(x)$ achieves passivity and further conditions must be checked determining the stability of such systems. To this end, according to [23], we need
\textbf{Definition 2:} The system (1) is zero-state detectable if $u(t) = 0$, $y(t) = 0 \forall t \geq 0$, implies $\lim_{t \to \infty} x(t) = 0$.

Using this definition, we are able to state stability results for passive systems from [21].

\textbf{Theorem 1:} Let the system (1) be passive with a scalar $C^1$ storage function $V$, then the following properties hold:

i) If $V(x) > 0 \forall x \neq 0$, then the equilibrium $x = 0$ of (1) with $u = 0$ is stable.

ii) If (1) is zero-state detectable, then the equilibrium $x = 0$ of (1) with $u = 0$ is stable.

iii) The feedback $u = -ky$, $k > 0$, achieves asymptotic stability of $x = 0$ if and only if (1) is zero-state detectable.

Hence, stabilizing passive systems is easy, as long as the zero-state-detectability is guaranteed. But this property implies that the system (1) is stable for $u = 0$. To remove this restriction one may use feedback to achieve passivity. This is known as feedback passivation [2] and will be exploited in the proposed design procedure in this article. However, application of feedback passivation is obstructed by two fundamental limitations with respect to the output $y$: i) the system under consideration must have relative degree one and ii) needs to be weak minimum phase. According to [21] these conditions can be relaxed for the systems introduced in the next Section.

We will use the above definitions to identify whether the systems under consideration, are passive. If so, we are able to find suitable control laws for coordinating these systems following [1, 3].

\textbf{B. System Description}

For our derivations, we consider strict feedback systems [12] of the form
\[ \xi_0 = a_0(\xi_0) + B_0(\xi_0)\xi_1 \]
\[ \xi_1 = a_1(\xi_0, \xi_1) + B_1(\xi_0, \xi_1)\xi_2 \]
\[ \vdots \]
\[ \xi_{k-1} = a_{k-1}(\xi_0, \ldots, \xi_{k-2}) + B_{k-1}(\xi_0, \ldots, \xi_{k-2})\xi_k \]
\[ \xi_k = a_k(\xi_0, \ldots, \xi_k) + B_k(\xi_0, \ldots, \xi_k)u, \]
where $\xi_m, u \in \mathbb{R}^p$, $a_m(\cdot) \in \mathbb{R}^p$, $B_m(\cdot) \in \mathbb{R}^{p \times p}$ and we need that $a_0(0) = 0$ and $B_m(\cdot)$ are invertible for all $m \in \{0, \ldots, k\}$.

For systems in the form of (2), a well known fact is, that the backstepping procedure [12] removes the relative degree limitation to feedback passivation by a recursive design procedure [21]. To this end, at each step an output is constructed such that the entire system is minimum phase using each $\xi_m$ as a virtual control input. Only in the last step the relative degree one requirement is satisfied and the real input $u$ can be specified. Thus, backstepping extends feedback passivation to systems having an arbitrary relative degree and we will make use of this in the synchronization control design for systems with dynamics (2).

\textbf{C. Graph Theory and Group Communication}

Key to coordination algorithms is information exchange within the group, and hence, the underlying communication network. This network can be modeled using graphs. We will introduce some basic definitions and notions important for the subsequent sections since the properties of these communication networks influence the stability of the coordinated motion of the group. We refer to [6] for a thorough treatment of graphs and their properties, and give a condensed introduction following [3].

\textbf{Definition 3:} A graph $\mathcal{G}$ is a finite set of elements $\mathcal{V}(\mathcal{G}) = \{v_1, \ldots, v_N\}$, the vertices of a graph, and a set $\mathcal{E}(\mathcal{G}) \subset \mathcal{V} \times \mathcal{V}$ called the edges of a graph. If for all $(v_i, v_j) \in \mathcal{E}$, $(v_j, v_i) \in \mathcal{E}$ as well, the graph is said to be undirected, otherwise it is called directed or Digraph. The in-degree of a vertex $v \in \mathcal{V}$ defines the number of edges incoming to this vertex, whereas the out-degree defines the number of edges outgoing from this vertex. A graph is balanced, if for each $v \in \mathcal{V}$ the in-degree equals the out-degree. Clearly every undirected graph is balanced.

\textbf{Definition 4:} A path of length $k$ in a Digraph is a sequence $v_0, \ldots, v_k$ of $k + 1$ distinct vertices such that for every $i \in \{0, \ldots, k - 1\}$, $(v_i, v_{i+1}) \in \mathcal{E}$ and a weak path is such that $(v_i, v_{i+1}) \in \mathcal{E}$ or $(v_{i+1}, v_i) \in \mathcal{E}$. A Digraph is fully connected if, for all $v_i, v_j \in \mathcal{V}$, $(v_i, v_j) \in \mathcal{E}$ and $(v_j, v_i) \in \mathcal{E}$. It is strongly connected if any two vertices can be connected by a path and it is said to be weakly connected if any two vertices can be connected by a weak path. An undirected graph can only be connected or disconnected, since there is no distinction between paths and weak paths. Concerning the interconnection topology the following assumption holds throughout the remainder of this article.

\textbf{Assumption 1:} The group of agents with individual dynamics (2) forms a balanced and static communication network which is strongly connected at all times.

\section{III. CONTROL DESIGN}

In this section, we will formulate the problem of synchronizing a group of agents and design cooperative controllers for systems with dynamics (2). We introduce a novel combination of passivity-based coordination and an alternative backstepping procedure. Based on a specific control Lyapunov function design for each step, the proposed controller provably stabilizes the aggregate motion of the group and synchronization is achieved.
A. Problem Statement

Suppose we have a group of $N$ agents each with dynamics given by (2) and with a communication topology satisfying Assumption 1. For synchronization, the goal is to drive each agent to a common final state \( \xi = [\xi_0^T, \ldots, \xi_k^T]^T \). Hence, the group is said to synchronize if

\[
\lim_{t \to \infty} \| \xi_m - \xi_n \| = 0, \quad \text{with} \quad m = 0, \ldots, k.
\]  

We use \( \xi_m \) to denote the variable \( \xi_m \) of the \( i \)-th agent whenever distinction between the agents is needed.

B. Design procedure

After stating the goal of our derivations, we will adapt the backstepping procedure to design a cooperative control law for each agent fulfilling (3). Since these controls only rely on the dynamics of the respective agent, the procedure will lead to fully decentralized controllers. Note that the following derivation is agent-based and holds for the whole group.

As a starting point, consider the first subsystem of the dynamics (2), that is

\[
\dot{\xi}_0 = a_0(\xi_0) + B_0(\xi_0)\xi_1.
\]  

For every agent, take \( \dot{\xi}_1 = a_0(\xi_0) \) as a virtual control input to stabilize this system such that there exists a function \( W_1(\xi_0) \geq 0 \), with \( W_1(0) = 0 \) and \( W_1 \leq 0 \) for all \( \dot{\xi}_0 \neq 0 \). Then, \( W_1(\xi_0) \) is considered as a Lyapunov function to show stability of (4). Simply choosing

\[
W_1(\xi_0) = \frac{1}{2} \xi_0^T \xi_0,
\]  

yields

\[
W_1(\xi_0) = \xi_0^T \dot{\xi}_0 = \xi_0^T [a_0(\xi_0) + B_0(\xi_0)a_0(\xi_0)].
\]  

Setting the virtual control to

\[
a_0(\xi_0) = -B_0^{-1}(\xi_0) [a_0(\xi_0) + \xi_0^T z],
\]  

results in

\[
W_1(\xi_0) = -\xi_0^T \dot{\xi}_0 \leq 0 \quad \forall \quad z \neq 0,
\]

giving asymptotic stability of (4). Note that this first step corresponds to the "exact backstepping" procedure, where complete cancellation is needed to obtain the desired behavior.

For the next step, introduce the error coordinate

\[
y_1 = \xi_1 - a_0(\xi_0)
\]

and extend system (4), applying this change of coordinates, to

\[
\dot{\xi}_0 = a_0(\xi_0) + B_0(\xi_0) [a_0(\xi_0) + y_1],
\]

\[
y_1 = a_1(\xi_0, a_0 + y_1) + B_1(\xi_0, a_0 + y_1)\xi_2 - a_0(\xi_0, y_1).
\]  

For the second step, take \( \dot{\xi}_2 = a_0(\xi_0, y_1) \) as a virtual control to stabilize this system such that there again exists a function \( W_2(\xi_0, y_1) \geq 0 \), \( W_2(0) = 0 \) and \( W_2 \leq 0 \) for all \( (\xi_0, y_1) \neq 0 \). Using

\[
W_2(\xi_0, y_1) = W_1(\xi_0) + \frac{1}{2} y_1^T y_1
\]

as a candidate Lyapunov function and computing the derivative with respect to the time \( t \) yields

\[
\dot{W}_2 = z^T [a_0 + G\alpha_0] + \xi_0^T \dot{y}_1 + y_1^T \dot{y}_1
\]

\[
= \dot{W}_1 + y_1^T \left[ B_1^T \dot{\xi}_0 - a_0 + a_1 + B_1 \alpha_1 \right].
\]

Here,

\[
\dot{W}_1 = W_1 |_{y_1 = 0} = -\dot{\xi}_0^T \dot{\xi}_0.
\]  

Next, we alter the standard backstepping design by setting the virtual control to

\[
\alpha_1 = B_1^{-1} \left[ a_0 - a_1 - B_0^T \xi_0 - B_0^T y_1 - y_1 - a_0 \right],
\]

such that

\[
W_2 = -(\xi_0 - B_0 y_1)^T (\xi_0 - B_0 y_1) - y_1^T y_1 = -\dot{W}_2 - y_1^T y_1 \leq 0,
\]

definition (\( \xi_0, y_1 \) $1$) with

\[
\dot{W}_2 = (\xi_0 - B_0 y_1)^T (\xi_0 - B_0 y_1).
\]  

Thus, asymptotic stability for (7) is achieved.

Proceeding step by step, simply introduce the error coordinates as

\[
y_r = \xi_r - a_{r-1}(\xi_0, y_1, \ldots, y_{r-1}),
\]

and extend (7) to

\[
\dot{\xi}_0 = a_0(\xi_0) + B_0(\xi_0) [a_0(\xi_0) + y_1],
\]

\[
y_1 = a_1(\xi_0, a_0 + y_1) + B_1(\xi_0, a_0 + y_1) [a_1 + y_2] - a_0
\]

\[
\ddots
\]

\[
y_r = a_r(\xi_0, a_0 + y_1, \ldots, a_{r-1} + y_{r-1}) + B_r(\xi_0, a_0 + y_1, \ldots, a_{r-1} + y_{r-1}) \xi_{r+1} - a_r,
\]

for \( r = 2, \ldots, k - 1 \). Then, using the candidate Lyapunov functions

\[
W_{r+1}(\xi_0, y_1, \ldots, y_r) = W_r(\xi_0, y_1, \ldots, y_{r-1}) + \frac{1}{2} y_r^T y_r,
\]  

choose the virtual control inputs \( \xi_{r+1} = a_r \) as

\[
\alpha_r = B_r^{-1} \left[ \alpha_{r-1} - a_r + B_{r-1}^T y_{r-1} + B_{r-1}^T B_{r-1} y_{r-1} - y_r \right].
\]  

This leads to

\[
W_{r+1} = -\dot{W}_{r+1} - y_r^T y_r,
\]

respectively, where

\[
\dot{W}_{r+1} = (\xi_0 - B_0 y_1)^T (\xi_0 - B_0 y_1)
\]

\[
- \sum_{n=2}^r (y_{n-1} - B_{n-1} y_n)^T (y_{n-1} - B_{n-1} y_n).
\]
In the last step, introducing the error coordinate \( y_k = \xi_k - \xi_{k-1} \) and applying this change of coordinates results in the extended system

\[
\begin{align*}
\dot{\xi}_0 &= a_0(\xi_0) + B_0(\xi_0) \left[ a_0 + y_1 \right] \\
\dot{y}_1 &= a_1(\xi_0, a_0 + y_1) + B_1(\xi_0, a_0 + y_1) \left[ a_1 + y_2 \right] - a_0 \\
&\vdots \\
\dot{y}_{k-1} &= a_{k-1}(\xi_0, a_0 + y_1, \ldots, a_{k-2} + y_{k-1}) \\
&\quad + B_{k-1}(\xi_0, a_0 + y_1, \ldots, a_{k-2} + y_{k-1}) \left[ a_{k-1} + y_k \right] - a_{k-2} \\
\dot{y}_k &= a_k(\xi_0, a_0 + y_1, \ldots, a_{k-1} + y_k) \\
&\quad + B_k(\xi_0, a_0 + y_1, \ldots, a_{k-1} + y_k)u - a_{k-1}. 
\end{align*}
\]

(15)

At this point, we are able to meet the relative degree restriction for feedback passivation: Considering \( y_k \) as the constructed (virtual) output, the real input \( u \) appears in the first derivative with respect to time of this output. Choosing \( V = W_k + \frac{1}{2} y_k^\top y_k \) as a candidate Lyapunov function gives

\[
\dot{V} = -\dot{W}_k - y_{k-1}^\top y_k - y_k^\top B_{k-1} y_{k-1} - a_{k-1} + a_k + B_k u + \tau. 
\]

(16)

In order to achieve feedback passivation in the sense of [2], choose

\[
u = B_k^{-1} \left[ a_{k-1} - a_k + B_{k-1}^\top y_{k-1} - B_{k-1}^\top B_k y_k + \tau \right]
\]

(17)

for each agent. Here, \( \mathcal{N}_i \) denotes the set of neighbors agent \( i \) receives information from. Concluding this derivation, we can state the following

Theorem 2: Given a homogeneous group of \( N \) agents with dynamics (2), interconnected through a static, balanced and strongly connected communication graph \( G \), synchronization, i.e.

\[
\lim_{t \to \infty} ||\xi_{2m} - \xi_{2m}|| = 0
\]

with \( m = 0, \ldots, k \), is achieved for the group with the controls

\[
u = \dot{B}_k^{-1} \left[ \dot{a}_{k-1} - \dot{a}_k + \dot{B}_{k-1}^\top y_{k-1} - \dot{B}_{k-1}^\top \dot{B}_k y_k \right] - K \sum_{i \in \mathcal{N}_i} \left( y_k - \dot{y}_k \right)
\]

(17)

for each agent, where \( \mathcal{N}_i = \{ j \mid (v_j, v_i) \in E(G) \} \).

Proof: Take

\[
U = 2 \sum_{i=1}^N \dot{V}
\]

as a Lyapunov function for the aggregate motion of the group, then its derivative with respect to time is

\[
\dot{U} = -2 \sum_{i=1}^N \dot{V} - 2K \sum_{i=1}^N \dot{y}_k^\top \left( y_k - \dot{y}_k \right)
\]

\[
= -2 \sum_{i=1}^N \dot{V} - 2K \sum_{i=1}^N \dot{y}_k^\top y_k + 2K \sum_{i=1}^N \dot{y}_k^\top \dot{y}_k.
\]

According to [3], for balanced graphs

\[
2K \sum_{i=1}^N \dot{y}_k^\top y_k = K \sum_{i=1}^N \dot{y}_k^\top y_k + K \sum_{i=1}^N \dot{y}_k^\top \dot{y}_k \leq 0,
\]

for every agent, and we conclude stability in the sense of Lyapunov for the aggregate motion of the group.

In order to prove synchronization, we need to employ the Krasovskii-LaSalle invariance principle [11]. Given a compact set

\[
\Omega \subseteq \mathbb{R}^p \times \mathbb{R}^p \times \cdots \times \mathbb{R}^p
\]

that is positively invariant with respect to (15), let \( U \) be a scalar \( \mathcal{C}^1 \) function such that \( U \leq 0 \) in \( \Omega \). We need to find the largest invariant set contained in \( \mathcal{M} = \{ z \in \Omega \mid U \equiv 0 \} \), where \( z = [ y_1^\top, \ldots, N^k \cdot y_1^\top, \ldots, y_k^\top ]^\top \). Due to the structure of \( U \) this means that \( \dot{z}_0 = \dot{B}_0 y_1, y_r - y_{r-1} y_r \) for \( r = 2, \ldots, k \) and \( y_k = \dot{y}_k \) for every agent of the group. On this set, applying the virtual controls (6), (10), (13) and the real input \( u \) from (16) together with (17) to each agent, the dynamics (15) reduce to

\[
\dot{z}_0 = (\xi_0 - B_0 y_1) \equiv 0,
\]

\[
y_1 = B_0^\top (\xi_0 - B_0 y_1) - (y_1 - B_1 y_2) \equiv 0,
\]

\[
\vdots
\]

\[
y_r = B_{r-1}^\top (y_{r-1} - B_{r-1} y_r) - (y_r - B_r y_{r+1}) \equiv 0,
\]

\[
\vdots
\]

\[
y_k = B_{k-1}^\top (y_{k-1} - B_{k-1} y_k) + \tau \equiv 0
\]

where again \( r = 2, \ldots, k \). We see that \( \mathcal{M} \) is an equilibrium set (invariant manifold) of (15) and therefore, is itself the largest invariant set contained in \( \Omega \). Due to the invariance principle we have that, all solutions starting in \( \Omega \) approach \( \mathcal{M} \) as \( t \to \infty \). From this it follows that \( y_k = \dot{y}_k, y_k - y_{k-1} = \dot{B}_{k-1} y_k, y_p = \prod_{k=0}^p \dot{B}_k y_k \) for \( p = k - 2, \ldots, 1 \), and finally \( \dot{\xi}_0 = \prod_{n=k-1}^0 B_n \dot{y}_k \). In this case, the operation \( \prod_{n=k-1}^0 \) denotes a left multiplication of succeeding factors \( B_n \). As all agents
are homogeneous $iB_m = iB_m \forall m \in \{0, \ldots, k\}$, and $i, j \in \{1, \ldots, N\}$ with $i \neq j$. Hence, we have $iX_0 = iX_0$, $y_0 = y_n$ for $n = 1, \ldots, k$, and synchronization in the sense of (3) is achieved, completing the proof.

**Remark 1:** The existence of a positively invariant set $\Omega$ is guaranteed by the construction of the Lyapunov function $U$. As $U$ is radially unbounded, one may take $\Omega = \{z \mid U(z) \leq c\}$ for any $c > 0$ [11], i.e., every level set of such Lyapunov functions describes a compact set.

**Remark 2:** Restricting this derivation to systems with $k = 1$ and $a_0(\xi_0) = 0$ results in an average consensus protocol similar to [17].

### IV. Example

Demonstrating the methodology on an appropriate example we choose the attitude coordination problem for a group of satellites modeled as fully actuated rigid bodies [5], [8], [19], [20].

Key to the description of the attitude dynamics is the chosen kinematic representation of the rotation with respect to an inertial frame. A well known fact is that some representations may cause singularities in the kinematic description due to non-uniqueness in the related mappings (see [22] for a complete overview). In order to avoid these problems in our approach, unit quaternions are used to represent the attitude kinematics for each system. A unit quaternion $q = [q_0, \eta]^\top$, where $q_0 \in \mathbb{R}$, $\eta = [\eta_1, \eta_2, \eta_3]^\top$, is defined such that $q_0^2 + \eta^\top \eta = 1$ holds. Following [22], the dynamics of each satellite modeled as a fully actuated rigid body is

$$\dot{q} = \frac{1}{2} \Xi(q) \omega,$$

$$\dot{\omega} = J^{-1} (u - \hat{\omega} J \omega),$$

where $\omega$ denotes the angular velocity, $J$ is the rotational inertia matrix and $u$ is the torque vector of the system relative to the body fixed frame, respectively, and

$$\Xi(q) = \left[ \begin{array}{c} -\eta^\top \\ q_0 I_3 + \eta \end{array} \right],$$

where $I_3$ denotes the $3 \times 3$ identity matrix. The operator $(\cdot)$ constructs a skew-symmetric matrix from a vector, such that

$$\hat{a} = \left[ \begin{array}{ccc} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{array} \right].$$

As fully actuated systems are considered, a torque about each of the body fixed axes can be applied.

For the derivation of the controller following our approach, note that the equilibrium of the given dynamics needs to be in the origin. Due to the unity restriction on the quaternion $q$ this is not true for (18). Thus, we need $\tilde{q} = q - [1, 0]^\top$ transforming (18) to the desired equilibrium. However, the dynamics of the system remain unchanged and we consider $\omega = \alpha$ as a virtual control and $\dot{W} = \frac{1}{2} \dot{\tilde{q}}^\top \tilde{q}$ as a candidate Lyapunov function for the kinematic equation of (18). Applying

$$\alpha = -2 \Xi^{-1}(q) \dot{q} = -2 \Xi^\top(q) \dot{q}$$

results in $\dot{W} = -\tilde{q}^\top \tilde{q} < 0, \forall \tilde{q} \neq 0$.

For the second (and in this case last) step we introduce $y = \omega - \alpha$ and obtain

$$\dot{q} = \frac{1}{2} \Xi(\alpha + y),$$

$$\dot{y} = J^{-1} \left[ \tau - (\alpha + y) J(\alpha + y) \right] - \dot{\alpha}.$$

Using $V = \dot{W} + \frac{1}{2} y^\top y$ as a candidate Lyapunov function and setting

$$u = (\alpha + y) J(\alpha + y) + J \left( \frac{1}{2} \Xi^\top + \alpha - \frac{1}{4} y + \tau \right),$$

Fig. 1. Plot of $\dot{q}$ over time for three agents with the control (20).
for each agent
\[ V = -\left( q - \frac{1}{2} \tilde{z} y \right)^T \left( q - \frac{1}{2} \tilde{z} y \right) + y^T \tau \]
\[ = -\frac{1}{4} \dot{\omega}^T \omega + y^T \tau \]
holds and passivity is achieved for each agent, respectively. Choosing the coupling control law as
\[ \tau = -K \sum_{j \in \mathcal{N}_i} ( y_j - y_i ) \]
one can conclude that attitude synchronization is achieved by the group of satellites using Theorem 2. For the complete computation of the input in (20),
\[ \dot{\alpha} = -2 \frac{d}{dt} \xi^T \dot{q} - \alpha - y = -2 \frac{d}{dt} \xi^T \dot{q} - \omega \]
is needed.

Results for a group of three satellites under the control (20) are shown in Fig. 1, where each component of the unit quaternion vector \( \dot{q} \) is plotted over time, respectively collected in one plot. The initial conditions for the satellites were randomly assigned and the coupling gain is \( K = 1 \). In this simulation the group forms a fully connected communication network.

The 3D movement of the group satellites is depicted in Fig. 2. Here, the situation at different time instances is shown until the final, synchronized configuration is reached (left most).

![3D animation of the attitude synchronization process.](image)

**Fig. 2.** 3D animation of the attitude synchronization process.

**V. CONCLUSION**

In this article we presented a method to derive passivity-based decentralized controllers synchronizing systems given in strict feedback form (2). We remove the relative degree obstacle to passivity-based coordination and by that extend existing results to systems having an arbitrary relative degree. To this end, we combined an alternative backstepping procedure with existing results on passivity-based coordination and achieve provably correct synchronization for a group of homogeneous agents.

As many mechanical systems are expressable in strict feedback form, future work will include the application of the proposed controller to such systems. Furthermore, we want to relax the conditions on the network topology holding in our proofs to include dynamic network topologies. Removing the weak minimum phase restriction to passivity-based coordination is still an open problem, as well.

**REFERENCES**